Investment to serve future consumption needs

-Trade theory applied on demographic challenges

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My first ideas about the pension debate, and the coming challenges on how to serve future consumption needs, appeared in discussions with researchers at the Frisch Centre, during a part time job there in the summer of 2001. When I later was introduced to trade theory at the University, the idea came up of applying this theory to demographic challenges. A somewhat ambitious idea for a thesis, some would say, and I am therefore grateful both to the Norwegian Institute of International Affairs (NUPI) who gave me a scholarship to study these questions and to my supervisor, Per Botolf Maurseth, who supported the idea from the beginning. He gave me full backing throughout the process, although he had some well-founded critical remarks. Thanks for constructive comments, good (and sometimes heated) discussions, and for offering much of your time!

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1. **Summary/Introduction**

The OECD countries have, during the four last decades experienced, a decline in fertility rates and an increase in life expectancy (UN, 2003). During the next 50 years the ratio of the number of workers to the number of elderly people will decline. The challenge is to meet the consumption needs. Many economists argue that a reformation of the present pension schemes is needed, and a transformation from the commonly used pay-as-you-go¹ (paygo) schemes to funding has been promoted, e.g. Modigliani et al (2000).

Poor countries have had a different demographic development, where many of them have high population growth rates that will result in much larger ratios of potential workers to the elderly than the OECD countries. There is also less capital equipment in poor countries than in the industrialized countries, which, in combination with the different demographic development, gives lower capital/labour ratios in poor countries.

This thesis discusses whether these differences between countries could cause interaction leading to a ‘win-win’ situation². The interaction could be trade or factor movements, or both. The theoretical starting point for the discussion will be the neoclassical framework, which predicts gains from interactions between countries with different factor endowments.

The simplest neoclassical³ production function with only two inputs, capital and labour, predicts that the return to capital will be higher in relative capital scarce countries than in capital rich ones. If this is true, one should observe a higher return to capital in poor than in rich countries. When capital is assumed to be mobile between countries, this could result in capital movements from rich to poor countries. Although many barriers of capital movements have been removed, these movements are not observed. This is sometimes referred to as the Lucas’ paradox, after the Chicago school economist Robert E. Lucas. In chapter 3 of this thesis this paradox is discussed within the Heckscher-Ohlin-Samuelson

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¹ Paygo is the system of most countries with a social security system today. The young, working generations are paying the benefits for the retired to day, while the coming working generation is going to pay the benefits for the people working today. Contributions from the working population is used to pay the pensions and thus to finance the consumption of the retired. Young workers contribute a fixed proportion of their labour income to the system, and the proceedings are divided into a lump sum among the retirees. Individuals who retire at mandatory age receive the full pension. The received benefits are decided from how many currently retired, how many currently working, and the worker’s income (GDP). If this is not the case, government uses other resources than pension contributions to finance benefits.

² A win-win situation is a situation where interaction will make all parts better off.

³ In this thesis the term ‘neoclassical’ will refer to the production functions with decreasing returns to scale with respect to each factor of production, in addition to a utility concept with the same properties; the utility is increasing, but the increase is decreasing as access to one good increases. The marginalist economics is sometimes regarded as covering the same aspects as neoclassical.
framework. It is concluded that a reformulation of the paradox may be necessary in a situation where labour intensive goods is less tradable than capital intensive goods. The possibility that labour intensive goods tend to be less tradable than capital-intensive goods should be added to the discussion of the Lucas’ paradox.

There is a debate on which pension system will be best suited given the demographic changes described above. In chapter 2 an introduction to relevant parts of the debate is given. In addition to providing the introduction to the discussion, this paper to a limited extent provides input to the debate itself. Chapter 2 also gives a description of the demographics in the OECD and the Norwegian Government Petroleum Fund.

Chapter 4 discusses saving for future consumption through foreign investment. As Norway has already invested parts of the petroleum income in foreign countries to cater for future consumption needs, the country provides an interesting case for the pension debate. The focus will be on whether or not non-traded goods can become traded, if money is spent to break down barriers to trade. This provides the rationale for using the general discussion of capital investments for future consumption to discuss the investments done by Norwegian authorities, when investing some of the oil revenues in a fund for future consumption. The fund in question is The Norwegian Government Petroleum Fund (PF), and is, with only few, small exceptions, invested in other OECD countries. This is called a world market portfolio investment, although not all markets in the world are invested in. The discussion will raise the question whether the PF should be invested in a world market portfolio, which is done today, or invested to break down barriers to trade in labour intensive goods from a poor country.

The consequences of the investment of the fund in the rich and the poor country will be discussed using the same neoclassical framework as in chapter 3. Two questions will be important in the discussion of whether or not the PF should be invested in the OECD countries:

1) Will the coming transformation of the pension schemes in OECD countries lead to lower rents on capital investments?

2) Can the PF be transformed into goods that fit the future consumption needs of an aging population?
1. The Challenging future

The OECD countries have a demographic situation that provides both current pension systems and future consumption needs, with a challenge. The ratio of the number of workers to the number of elderly in the population is decreasing. The need for service provision for the old is growing, leading to a higher demand for labour in the service sector. On the other hand, as shown below, the total supply of labour for the economy is decreasing. If the capital stock is fixed or grow at a positive rate, the capital/labour ratio in these countries will grow.

Poor countries, outside the OECD, also face a challenge, concerning how to meet future consumption needs. The demographic situation in these countries is different from that of the OECD; the population growth results in a much smaller rate of elderly to the working population, although the rate increases as shown in diagram 1. In this paper, lack of capital is seen as the main indicator of poverty at a national level. The flow of capital from abroad, in addition to the domestic savings, will probably be lower in many poor countries than the population growth. If the savings rate and the rate of inflow of foreign capital are constant, the result is a decreasing capital/labour ratio.

The neoclassical models of production, predict that differences in capital/labour ratios between countries should result in trade, migration or capital movements. In this tradition, one or more of the following effects of current demographic trends are predicted:

- Capital will move from rich to poor countries where the rate of return is higher
- Labour will move to the rich countries where the wage is higher
- Trade will occur; capital intensive goods will be sent from the rich countries to the poorer ones, while labour intensive goods will go in the opposite direction
- More capital investments in one region, all other variables being constant, will lead to a decrease in the return to capital in that same region

The first three predictions are considered in chapter 3, while the fourth is discussed in Chapter 4. In this chapter, a description of the demographics of the OECDs will be given, by use of examples from Norway. An introduction to the debate on different pension systems and a description of the Norwegian petroleum Fund, which might be an instrument for
Norway to handle the forthcoming challenge to meet future pensioners’ consumption needs, are given.

2.1 The demographic situation for the OECD

The OECD countries are experiencing a decrease in the ratio of the number of workers to the number of elderly. What may be more surprising is that also developing countries will experience such an increase, despite the relatively high population growth rates in many of these countries. The increase will be moderate in the least developed countries, and they will not in the next fifty years to come reach the nowadays level for more developed regions (UN, 2003). Diagram 1 shows this tendency. For the more developed and less developed regions the rate of people above 60 to total population will almost triple until 2050.

Diagram 1 Number of people above 60 years to the total population

When the rate of elderly to the total population increases national public pension schemes will experience an increased burden to pay pensions. The increased burden is caused by:

- Higher claims due to the increased number of retired people
- Fewer people to pay the claims

Diagram 2 shows the increased claims while diagram 3 shows the decrease in number of workers to pay the pensions, for the case of Norway⁴.

**Diagram 2 Pension benefits required in Norway*\(^\text{(*)}\)**

(*Numbers in billions NOK.

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⁴ Tables drawn on the basis of SSB predictions. Appendix A describes their assumptions.
As can be seen from diagram 3 the predicted percent of the population that is working will decrease, approaching 68.5% for Norway in 2040. Comparing the Norwegian example where the ratio of workers to total population is approximately 69.5% to the mean value of the ratio of elderly to total population for more developed region shows that the demographic changes in Norway are less dramatic than the general level for more developed nations.

It is evident, though, that the increased number of pensioners in Norway will lead to higher expenditures needed to pay pensions, and therefore a bigger burden on the working part of the population, at least as long as no reform of the pension system takes place. The Norwegian pension benefits are increased as the real wage increases. Productivity growth does not therefore provide a solution to this problem.
2.2 The pension debate

It is widely accepted that the demographic development described in the last section constitutes a substantial challenge to the pension systems in all OECD countries. There is, however, no unity as to whether or not the system should be changed from the most commonly used paygo system to a system based on pre-funding. Funded systems are different from paygo in the sense that each generation are supposed to build up piles of money or assets to pay for own pensions when they retire. Modigliani F., M. Ceprini and A.S. Muralidhar (2000) argue that one should replace the current public paygo system in most countries today, with a fully funded scheme. The argument is that funding increases saving, and saving in financial assets give a higher rate of return than the sum of population and productivity growth. This will then lead to lower contributions needed in the future. They propose a shift from the paygo system to a fully funded system for the United States. They argue that the fully funded system is more cost effective in the sense that it requires smaller contributions for a given set of benefits. Different fixed rents on capital are assumed in their study. When 5% rent is assumed, they argue that the required contribution in the funded system would be approximately 1/3 of that of the paygo system. In their opinion a funded system is more stable and financially resilient, resulting in a large accumulation of assets and thus makes a contribution to national saving. Their proposal is that all participants’ contributions be invested in one common highly diversified portfolio consisting of a share of US’ domestic production, that they call the US’ market portfolio.

Others, e.g. Brooks (2000), argue that the relatively high return on capital that we have seen in the last century in the United States can not be seen as constituting common rule for future predictions. The U.S. has in recent years experienced declining rates of return and the capital market of Japan, and other countries have experienced recessions. Robin Brooks (2000) uses an overlapping generation model to show that changes in the age distribution have significant impact on financial markets. He demonstrates that returns on savings for members of large birth cohorts (e.g. 1945-50) will be substantially lower than for other, smaller cohorts.

5 Two main groups of funded schemes are the defined benefit schemes and the defined contribution schemes, respectively. The defined benefit system is a state-led or private system where, as the name suggests, the benefits received when retired is defined. The contributions paid when working are then figured from expected returns. The defined contribution system is either state-led or private, contributions are decided and benefits will be counted from returns of the system.

6 In a paygo scheme, the value growth of the system is the sum of population growth and productivity growth
Whereas Brooks is arguing within a closed economy, Holzmann (1999) is including the World as a whole in his statement. Without further discussion he concludes: “the non-synchronised ageing between developed and developing countries could provide higher returns for the one and higher capital formation for the other”.

The next chapters contain an analysis as to whether the statement by Holzmann, as is evidently the case for Brook’s arguments, can be shown to be valid in the context of a neo-classical tradition. The possibility that the non-synchronised ageing between the developed and developing countries could cause integration that results in higher returns to developed countries and higher capital formation for developing countries than in the absence of such integration will be the starting point for most of the analysis in this thesis, and we leave it for the discussion in the next chapters.

In addition to the claim that return to capital may fall, Modigliani et al’s view has been met by another main argument. Orzag and Stiglitz (1999) and Nicholas Barr (2000) stress the centrality of output. They make the point that pensioners are not interested in piles of money, but consumption.

Barr’s illustration goes as follows: If pensioners build up piles of money for future consumption, the desired pensioner consumption in the next period will exceed desired saving by workers. This results in price inflation and reduced purchasing power for pensioners. If, on the other hand, pensioners have accumulated non-money assets, the pensioners’ desired asset sales exceed desired asset purchases by workers. This results in reduced asset prices and hence reduced pensions.

Barr (2000) states that in either outcome, the pensioners do not get what they expect, and the funded schemes face similar problems to paygo. The reasons are the same: shortage of output. Further he claims that the only difference is that the process is less transparent in funded schemes, and that a reason for advocating the funded system might be that politicians like bad news be seen to arise through market outcomes rather than political decisions.

The only way a transition to a funded system can meet the challenges of today’s pension schemes is then, according to Barr (2000) and Stiglitz and Orzag (1999), through an increase in output.

The question then is whether or not a transition to a funded system can increase output. To answer this question, two new questions have to be answered:

- Does pre-funding increase saving?

---

7 The two arguments are related, and can be said to be only two ways to express the same point, in a closed economy.
• Does saving increase output?

The answer to the first question might be yes or no depending on whether or not savings for pensions crowd out other savings. If the theory of strong Ricardian equivalence holds, pre-funding will not increase savings. Barr (2000) argues that there is no robust evidence that a switch to a funded system increases saving in any country except for the US, and he also claims that the US’ evidence is not very robust.

Whether or not saving increases output is a broad debate that touches the difference between theories with demand side focus and supply side focus respectively. The Keynesian approach that has a demand side focus, argues that increased saving will reduce output, while the neoclassical approach, with supply side focus, will argue that increased savings increase output in the coming period. As already mentioned the framework used in this thesis is the neoclassical and therefore we will assume that saving increases output in an economy.

An issue that is not explicitly discussed in any of the two papers, Barr (2000) and Stiglitz and Orzag (1999), is the difference between domestic and foreign pre-funding. Whether or not domestic pre-funding will meet the challenges of future consumption needs will for certain relay on whether or not output increases. If a pension fund is invested in foreign markets, though, future consumption of pensioners does not necessarily relay on domestic output. If pensioners have preferences also for traded goods, consumption can be higher than production. This issue will be discussed in chapter 3 and 4, and therefore the points made in this section will relay on domestic pre-funding.

Orzag and Stiglitz stress that a discussion of whether or not a funded system is better than paygo is different from the discussion of whether or not a transition to a funded system is preferable to paygo. The transition to a funded system will lead to what is called a double burden for one generation. This means that one working generation has to pay for both the pensioners when they still work and their own pension after retirement. That is one of the reasons why Norway is an interesting example: The Norwegian Petroleum Fund, which will be returned to in next section, could be used to pay one of the burdens. If it is accepted that the Petroleum income should belong to all future Norwegian generations, that transition burden is then paid by all future generations.
2.3 The Norwegian Petroleum Fund

The Government Petroleum Fund (PF) was established in 1990. The act relating to the PF is Act of June 22, 1990 no 36. The two main purposes of the Fund are the following (Finansdepartementet, 1990):

- Act as a buffer to smooth short term variation in the oil revenues
- Serve as a tool for coping with the financial challenges connecting to an ageing population and the eventual decline in oil revenues, by transferring wealth to future generations

The first part of the last purpose is the focus in the rest of this thesis, namely the aim of using the PF to serve the consumption needs of the ‘boom’ of elderly retiring.

Although the Fund was established in 1990, no transfer was done until 1996. The first transfer to the Petroleum Fund was made for the 1996 fiscal year and amounted to NOK 2 billion. Today the income of the Fund is defined as the government’s net cash flow from petroleum activities plus the return on the PF’s capital and net financial transactions related to petroleum activities. The expenditures of the Fund are the transfers to the fiscal budget to finance the non-oil budget deficit.

At the end of 2002 the Fund amounted to NOK 609 billion. The return on the fund is given in box 1.

**Box 1 Returns on the Norwegian Petroleum Fund**

<table>
<thead>
<tr>
<th></th>
<th>1997</th>
<th>1998</th>
<th>1999</th>
<th>2000</th>
<th>2001</th>
<th>2002</th>
<th>Total period</th>
</tr>
</thead>
<tbody>
<tr>
<td>Return</td>
<td>7.28%</td>
<td>8.16%</td>
<td>10.94%</td>
<td>0.34%</td>
<td>-3.72%</td>
<td>-6.57%</td>
<td>2.53%</td>
</tr>
</tbody>
</table>

(*) Numbers are taken from the Norwegian Central Bank’s yearly report 2002 (Norges Bank 2002).

The Ministry of Finance is the manager of the Fund. In the Regulation on the Management of the Government Petroleum Fund of October 1997, the Ministry of Finance has delegated the responsibility for the operational management of the PF to the Central Bank of Norway. The Bank has guidelines from the Government and the Ministry of Finance, and the government informs the parliament regularly. The Ministry of Finance
decides a benchmark portfolio and risk limits, while the central bank carry out the investment strategy, have the responsibility of risk management, accounting and reporting and offer professional advice to the ministry.

It is said in the regulation that the total fund has to be invested abroad. The arguments for this is the need to maintain and protect the fiscal budget as a management tool, the need for a diversified industry structure, i.e. avoid “Dutch disease” and the need to stabilize the Norwegian economy by avoiding pressure on the exchange rate and decrease in interest rates that might have happened if the fund was invested in domestic markets.

Originally, the Fund was invested in line with the Central Bank’s foreign exchange reserves, i.e. in low-risk interest bearing securities. Today the fund also holds equities. Two arguments for this have been claimed:

- Risk must be weighted against the expected returns. Stocks entail a higher risk of short-term fluctuations in market value than bonds, but have historically provided a higher returns over time.
- The returns on equities and bonds do not normally move together

It is claimed that the Fund is well diversified although some parts of the world are not included in the benchmark portfolio. An overview picture of the benchmark portfolio is shown in diagram 4.

Diagram 4 Overview of the benchmark portfolio

The PF is with only small exceptions invested in the OECD countries. Brazil, Singapore, Hong Kong, and Taiwan are the countries outside the OECD that is included. They, altogether, only share a few percent of the equity investments and less than one percent of the investments in bonds (Norges Bank, 2002).
3. Why do investments take place in capital abundant rather than in capital scarce countries?

The waste of labour is by no means confined to rich industrial countries. It is considerably greater in poor agrarian countries. If the principles of international division of labour are to be applied, labour must either be transported towards capital (emigration), or capital must be transported towards labour (industrialisation). From the point of view of maximising the world income, the difference between these two ways is one of transport costs only, and may be assumed to be negligible. Emigration and resettlement would, however, present so many difficulties in immigration areas (and in emigration areas) that it cannot be considered feasible on a large scale. A very considerable part of the task will have to be solved by industrialisation. (Rosenstein-Rodan, 1943)

Rosenstein-Rodan’s paper on development for Eastern Europe expresses the idea of externalities on the demand side. The theory used in this chapter follows the neo-classical tradition and no externalities are assumed. The description of how the capital should move to poor countries, however, fits with the predictions from neo-classical models.

Neo-classical models of production and international trade predict that return on capital is higher in capital scarce countries than in more capital abundant countries. This holds under the assumption that there is not perfectly free trade with no specialisation, as in the HOS framework, where factor price equalization occurs.

Economists following the neo-classical school have, as already mentioned, asked why capital does not flow from rich to poor countries, as the models predict. This is earlier referred to as the Lucas’ paradox.

In this chapter the neo-classical production function and some of its predictions are presented. Possible explanations to the Lucas’ paradox are given with reference to some research in this area. Further, at the end of the chapter, the paradox is discussed within the Heckscher-Ohlin-Samuelson framework for international trade.
3.1 The neo-classical production function

The simplest general production function in the neo-classical tradition is given by:

**Box 2 The neoclassical production function**

\[
(1) \quad Y = F(K, L) \quad F_i' > 0 \quad F_i'' < 0 \quad \text{where} \quad i = K, L \quad \text{and} \quad i \neq j \quad F_{ij}'' > 0
\]

If Y measures a country’s total production, and if L and K is the number of labour units and capital units respectively, the GDP will increase as total labour force and/or total amount of capital, increase. As one of the amounts of inputs increases, though, the increase in total GDP increases less and less.\(^8\) If the same technology is assumed to be present in all countries, the same production functions will also apply to each of them. This will imply that the relative capital scarce country has a higher rate of return on capital than the more capital abundant country\(^9\), except the case with HOS trade that is mentioned earlier. In this simple framework, there are only two inputs: physical capital and labour. A rich country is characterised with a higher capital to labour ratio than a poor country, which implies that poorer countries have a higher return to capital than richer ones.

Although rich countries have more capital relative to workers than poor countries, large capital flows from richer to poorer countries do not take place, as the Neoclassical

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\(^8\) For the case of completeness, the Inada conditions could be added to the definition of the neoclassical production function. They state that if one of the factors approaches the amount of zero, the marginal productivity of that factor approaches infinity, and when one of the factors approaches the amount of infinity, the marginal productivity of that factor approaches zero.

\(^9\) An example of a neo-classical production function is the Cobb-Douglas function \(Y = AK^\alpha L^\beta\). L is labour in production, K is capital used in production, while Y is the Gross Domestic Product. Both \(\alpha\) and \(\beta\) are parameter values between zero and one. The marginal product of capital equals the rent on capital: \(\frac{\partial Y}{\partial K} = \alpha AK^{\alpha-1}L^\beta = r\)

where \(r\) is the rent on capital. The rent on capital is higher the higher \(L\) and the lower the higher \(K\).

\[
\frac{\partial^2 Y}{\partial K \partial L} = \frac{\partial r}{\partial L} = \alpha \beta AK^{\alpha-1}L^{\beta-1} > 0 \quad , \quad \frac{\partial^2 Y}{\partial K^2} = \frac{\partial r}{\partial K} = (\alpha - 1)\alpha AK^{\alpha-2}L^\beta < 0
\]
Production Function suggests. This is referred to as the Lucas’ paradox, named after the Chicago School economist, Robert E. Lucas. Lucas himself incorporated human capital, and pointed at the role of externalities and imperfect capital markets. He also suggested that the possibility of wage increase in the pre-colonies might have been the reason why the colonial powers did not invest in the colonies (Lucas, 1990). Other explanations have been given, e.g. Rodrik (1991), who points to the role of risks and policy uncertainty in developing countries.

Risks, other production functions, and three of the four explanations given by Lucas himself, monopsony in the labour market, differences in human capital, and externalities are described below.

### 3.1.1 Risks

Dani Rodrik (1991), among others, points to the risks and policy uncertainties in developing countries. He claims that they work as a tax on investments, and even modest risks and uncertainties, can act as a large tax on capital investments. A simple model of possible risks of expropriation, or conflict that destroys the capital in the region, is explained in the following. In the case of conflict or expropriation, all capital is destroyed.

Rodrik, however, allows for some capital to return to the investor also in the case of a conflict, but at an entry cost.

#### 3.1.1.1 A simple model with risks of expropriation

- \( q \) – probability of expropriation
- \( r^* \) - the capital rent on the world market
- \( K \) – the amount of capital invested
- \( r \) – the rent on capital in the developing country

A simplified model would be a world market where the risks to investments is set to zero, and another market, namely a poor country with a risk for a conflict, which destroys all the capital. A risk neutral investor would be indifferent as to where to invest, if the expected returns in the two areas are equal.
The investor is thus indifferent if:

\[(2) \quad r^* = (1 - q)(1 + r) - 1 = r(1 - q) - q \]

Assume that the rent on the world market is 10%, and the probability of risk is 0.1. For the investor to be indifferent, the rent on capital in the developing country has to equal:

\[(3) \quad r = \frac{r^* + q}{1 - q} = \frac{0.11}{0.9} = 0.122 \]

A 12.2% rent on capital will in this case make the investor indifferent. If the risk of expropriation is larger or if the investor is risk averse, the rent in the developing country has to be even higher.

If we accept that developing countries tend to have higher risks and policy uncertainties, for example due to lack of institutional capacity and regulation, this might be an explanation to the Lucas’ paradox.

### 3.1.2 Monopsony in the labour market

Lucas is himself sceptical to the risk argument being the only explanation to the Lucas paradox. Before 1945 most of the poor countries were subject to European colonial political and economical leadership. The argument of risks and political uncertainty would then not be valid, as the colonial powers were themselves instrumental in influencing the risks. Even at that time, however, movement of capital between the rich countries and the colonies was very limited. Lucas suggests that investments would increase wages in the colony and a profit maximising colonial power would therefore under-invest.

Moene (1998) set up a model discussing the possibility that this mechanism may also be present today. The monopsonist is no longer the colonial power, but the elite in the developing countries. His model is described below.

There are two sectors of production, G and F. G uses labour only in production, while F uses both labour and capital. The total output in the economy is:

\[(4) \quad Y = G(L_G) + F(L_F, K) \]
G has decreasing returns to scale, and Moene interprets this as natural resource extraction. F is called industry and is assumed to have the same properties as the neoclassical production function in equation (1). Equation (7) shows that there is a constant amount of labour in this economy, and the whole labour force is fully utilised.

What he describes as the elite in the poor country can decide the size of K. When this is decided, the wage is dependent on equilibrium in the labour market. Equilibrium in the labour market is characterised with equal marginal productivity of labour in each sector of production, and wage equal to this marginal productivity:

\[
g'(\bar{L} - L_F) = F_{L_F}^{\prime}(L_F, K) = w
\]

From equation (6) we know that the cross derivative of production with regard to labour and capital in the F sector is positive. This implies that the higher the amount of K in the economy, the higher the wage:

\[
w = w(K) \quad w'(K) > 0
\]

The employment in the G sector will increase as w goes down, i.e. the less K the higher \(L_G\). This can be seen from (5) and (8).

The opportunity cost of investing in the poor country is the world market rent on capital, r.

The profit of investing in the poor country is thus given by:

\[
\pi = Y - w(K)\bar{L} - rK
\]

The economic efficient amount of capital is given in equation (11) where the marginal productivity of capital equals the world market rent.
Maximising (10) however, gives the optimal amount for the colonial investor:

\[
\frac{\partial \pi}{\partial K} = F'_K(L_F, K) - w'(K)\bar{L} - r = 0
\]

\[\Rightarrow\]

\[
F'_K(L_F, K) = w'(K)\bar{L} + r
\]

The marginal productivity of capital has to be higher than the world market return, for an investor to invest in the poor country.

Given that a monopsonist elite as assumed above is present in poor countries, this model explains the Lucas’ paradox.

3.1.3 Human capital

Another point made by Lucas, is the role of human capital in the production. If human capital, \( H \), is incorporated in the production function, and capital and labour are assumed to produce more effectively the higher the \( H \), this explains why capital is invested in rich countries rather than in developing countries. One example of a production function with this property is the Cobb-Douglas production function:

\[
F(L, K, H) = K^\alpha H^\beta L^{1-\alpha-\beta}
\]

\[\alpha > 0 \quad \beta > 0 \quad \alpha + \beta < 1\]

The rent on capital will be given by:

\[
r = \frac{\partial Y}{\partial K} = \alpha K^{\alpha-1} H^\beta L^{1-\alpha-\beta}
\]
We see that the rent on capital is higher the higher the H. This will be true for all production functions with positive cross derivative with regard to capital and human capital. In the Cobb Douglas case the cross derivative is equal to:

\[
\frac{\partial^2 Y}{\partial K \partial H} = \frac{\partial r}{\partial H} = \beta \alpha K^{\alpha-1} H^{\beta-1} L^{1-\alpha-\beta} > 0
\]

The fact that rich countries tend to have higher human capital may therefore be another explanation to the Lucas’ paradox.

3.1.4 Externalities

Positive externalities from capital or human capital are also used as explanations to the Lucas’ paradox. Lucas pointed at externalities from human capital. One possible way of showing this in the neoclassical production function is by including a technology parameter A, taken as given for each producer in the economy. If A is assumed to be a function of the total amount of capital (or human capital) in the economy, the externalities can be expressed in the following way:

Assume each producer in the economy has a production function equal to:

\[
Y_i = AK_i^{\alpha} L_i^{1-\alpha}
\]

Where \( K_i \) and \( L_i \) is capital and labour, respectively, in firm i.

The total production in the country will then be all the individual firms’ productions added together:

\[
Y = \sum_{i=1}^{n} Y_i = \sum_{i=1}^{n} AK_i^{\alpha} L_i^{1-\alpha} = A \sum_{i=1}^{n} K_i^{\alpha} L_i^{1-\alpha}
\]

If all the n producers are of equal size, the total production is:

\[
Y = AK^{\alpha} L^{1-\alpha}
\]
An externality can be introduced by endogenizing $A$:

\[ A = \left( \frac{K}{L} \right)^\gamma \]

The production function for the whole economy can then be expressed as:

\[ Y = K^{\alpha + \gamma} L^{1-\alpha - \gamma} \]

where $K^\gamma$ expresses the positive external effect from capital.

If an external effect of capital is present this will counteract the earlier point made, that capital scarce countries will have higher return on capital. Seen separately, the externality will cause higher return to capital in already capital abundant countries. The two effects will have opposite effects of the initial amount of capital in the economy, and the total effect on the rent of capital of being capital abundant could be negative, as without the externality, or positive.

The return that the individual investor expects, while ignoring the external effect, $r^i$, is given by:

\[ r^i = \frac{\partial Y}{\partial K} = \alpha A K^{\alpha - 1} L^{1-\alpha} > 0 \]

When recognising that $A$ is a positive function of $K$, the amount of $K$ in the country will then have an unknown negative effect on $r^i$. Using the above example of an $A$ function the derivative of $r$ with respect to $K$ is given by the second derivative of $Y$ with respect to $K$:

\[ \frac{\partial r}{\partial K} = \frac{\partial^2 Y}{\partial K^2} = (\alpha + \gamma)(\alpha + \gamma - 1)K^{\alpha + \gamma - 2} L^{1-\alpha - \gamma} \]

Which is less, equal to, or larger than zero depending on whether or not the sum of the parameter values, $\alpha$ and $\gamma$, is larger than 1.

One possible explanation to the Lucas’ paradox is that capital in itself has a positive external effect on the total production in an economy.
3.1.5 Other production functions

Unquestionably, the theoretical neoclassical production function does not fit all the actual production procedures. For example, it might be the case that not all factors of production are fully utilised, as is assumed in the neoclassical framework.

The example that will be given in this section is a production function where all factors of production are fully utilised, but the productivity of each worker is positively related to the productivity of each worker. The so-called O-Ring production function explained in Kremer (1993) is given by:

\[ Y = K^n \left( \prod_i q_i \right) nB \]

Where \( B \) is a constant, \( n \) is the number of tasks in the production process, and \( q \) is the quality of the work done in each task. The similarity with the neoclassical production function is not difficult to discover; the cross derivative of production with regard to \( K \) and \( n \) is positive. The difference, however, is that the better quality each worker has, or better the quality in each task of production, the better the marginal productivity from each worker’s or task’s quality. This is the opposite in the neoclassical function. An example of the kind of production process expressed in the O-ring function is one along an assembly line.

If \( q \)’s interpretation is human capital, the similarity between this function and the neoclassical with human capital incorporated, is that higher human capital in the rich countries can explain why capital does not move from rich to poor countries. The effect of differences in human capital is much larger, though, in the O-Ring function.

If there were some positive connection between \( q \) and \( K \), for example because workers that are used to working with capital equipment and machinery does that in a more effective way than those not used to it, this explanation to the Lucas’ paradox would be even stronger.
3.2 The Heckscher-Ohlin-Samuelson framework\textsuperscript{10}

The Heckscher-Ohlin-Samuelson (HOS) trade theory is a framework within the neo-classical tradition. The production in each country consists of the output from two sectors of production. In the following these will be referred to as C and S, respectively. The production function for each of the two sectors is given by:

\begin{align}
S &= F_S(K, L) \\
C &= F_C(K, L)
\end{align}

The production functions satisfy the neo-classical assumptions presented in the previous section. They are also characterised by constant returns to scale (CRL). The goods produced in each sector will not be given any interpretation yet; the only necessary assumption is that the goods produced in one sector have the same production technology, and that consumers have common preferences for all goods in one sector. The two sectors, however, have different production technologies, and the C sector is assumed to be most capital intensive in production.

Two countries will be considered, and we will divide the world into developed and developing regions. One country will be called O, symbolising the OECD countries, while the other country will be called D, symbolising the group of developing countries. The two factors of production, labour and capital, are perfectly mobile between sectors inside one country and perfectly immobile between countries. Homothetic preferences are assumed, i.e. the income elasticity is equal to one\textsuperscript{11} and consumers in both countries are assumed to have similar paths of homothetic preferences.

\textsuperscript{10} A more formal outlay of the HOS trade theory is given in appendix B
\textsuperscript{11} This will imply that all consumers, although at different income levels, will have preferences for the same basket of goods. There is thus no “Engel effect”, i.e. that the richer people use larger shares of their budgets on luxury goods than poorer people do. See for example Mehlum (2003) for other preference assumptions. Homothetic functions are a monotone incr
It will be assumed that the OECD countries are relatively well equipped with capital than the developing regions. For the purpose of illustration, assume that no barriers to trade are present. The result will then be that the OECD country will export the capital intensive good and import the labour intensive good, while the opposite will be true for the developing country.

Diagram 6 below shows the production possibility set (PPS) for the poor country and the relative price line, p, in autarky for this country. The PPS is concave because the two goods have different production technologies and the second derivatives of the production functions are less than one with regard to capital and labour respectively. We will later see that the slope of this country’s PPS is different from the slope of the rich country’s PPS; the poor country can produce more S goods relatively to C goods due to the fact that the country is a capital scarce country. The intersection with the axes shows the output of this country measured in S and C goods respectively.

The indifference curve drawn is one for a “representative consumer”. The assumption necessary to draw this curve is that it is possible to aggregate consumers in such a way that all consumers’ preferences could be illustrated by one set of indifference curves. The assumption of homothetic preferences ensures this. The indifference curve that is tangent to the PPS will ensure efficiency both in consumption and in production; a point inside the PPS will not use all that is produced (unexploited gains) and a point outside this is not possible because of the production possibilities.

easing function of a function that is homogenous of degree one. They are illustrated by drawing indifference curves that all have the same steepness in each point that make a linear curve that crosses origo.
Diagram 5 Production possibility set and autarky prices for the poor country

Diagram 5 shows the production possibilities (PPS) for the poor country; it is concave, indicating that there are substitution possibilities between the factors of production in the two sectors. The PPS’s intersection with the horizontal and vertical axes gives the production in the poor country given in $C$ goods and $S$ goods respectively. All points on and inside the PPS are possible production points, but optimality in production will imply that the production point is on the PPS. The consumers’ preferences can be shown by convex indifference curves, indicating that consumers prefer some of both goods compared to just consuming one of the goods. The preference direction is northeast in the diagram, indicating that more goods are always better, all other things equal. The optimal possible consumption point will then be the one where one of the indifference curves is tangent to the PPS. This indifference curve is called $U$ in the diagram. The slope in this optimal point for both consumption and production, gives the negative of the relative price level, $-\frac{P_C}{P_S}$.

The OECD is relatively, and absolutely, more capital abundant, and therefore the PPS of the OECD less steep, for any given value of $C$. Diagram 6 shows one possible illustration of the PPS for the OECD and the relative price line in autarky for this country.
Diagram 6 Production possibility set and relative prices in autarky for the OECD

By putting these two figures together one can find the relative price line for the world-integrated equilibrium, i.e. when all gains from trade are exploited. This is done in diagram 7.

The U and the PPS has the same interpretation as in Diagram 3.1. The price line, P, has the slope of the negative of the autarky price level in the OECD. The PPS of the OECD country is less steep than the one for the poor country, indicating that the OECD is more capital abundant and resting on the assumption that the C goods are relative more capital intensive in production.
Diagram 7 Integrated equilibrium, world market relative prices

The PPS in diagram 7 is constructed as if the two countries were one common country, and the factors of production in the whole world give the production possibilities drawn here. With perfectly free trade, the optimal consumption - and production point is the tangent point between \( U \) and the world’s PPS. The slope of \( P^* \) gives the negative of the relative world market prices, \( \frac{P_C}{P_S} \).

Because of the assumption of homothetic preferences, it is possible to draw the indifference curve for a representative ‘world consumer’ that is tangent to the PPS of the world.

We see that the relative price line for the integrated world market is not as steep as in the poor country, but steeper than the price line of the rich country. A more formal description of the integrated equilibrium compared to the autarky situations is given in the appendix to this chapter.
Box 3 Assumptions in the HOS framework

HOS assumptions:
- 2x2x2 model; two countries, two goods and two factors of production
- Constant returns to scale (CRS) production functions, the same technology in both countries
- No factor intensity reversals
- No specialisation
- Homothetic preferences, equal in both countries
- Factors of production perfectly mobile between sectors and perfectly immobile between countries
- No barriers to trade in commodity markets

The following section will adopt all the assumptions listed in box 3, while section 3.2.2 relaxes the two last assumptions.

3.2.1 Free trade in Commodity Markets

The situation considered in this section is one with immobile factors of production while the commodities are perfectly mobile and can be traded without costs. As long as there is no perfect specialisation in the post-trade equilibrium, the relative commodity prices after trade will be equal in both countries and lie in the interval between the two pre-trade autarky prices. Assuming that the poor country is relatively labour abundant, this will be given by the relation:

\[
\left( \frac{P_S}{P_C} \right)_{\text{Post-trade}} \in \left( \frac{P_S}{P_C} \bigg|_D, \frac{P_S}{P_C} \bigg|_O \right)
\]

Top-print A indicates the autarky prices and footprint O and D represents the developing- and the OECD countries, respectively. The autarky price levels are shown in diagram 8. 8a shows the possible aggregate production and consumption levels, while 8b shows the per
capita levels of production and consumption respectively. The relative price levels, i.e. the slope of the O and E curves, are equal in the two diagrams.

**Diagram 8 Production and utility in O and D**

Diagram 8a shows the PPS for the OECD and the poor country in the same diagram, and the two autarky price lines. Diagram 8b shows the production possibilities per capita. We see that the poor country consumer reaches a lower utility level \( u_2 \) than the OECD country consumer \( u_5 \). This gives a justification for calling it the poor country.
In both diagrams the slopes of the two straight lines named O and D show the relative price levels in the poor and the rich country respectively. The slope of the O-line is equal to

\[-\left(\frac{P_s}{P_c}\right)_O^A\]

and the slope of the D-curve is equal to

\[-\left(\frac{P_s}{P_c}\right)_D^A.\]

The additional information we get from the per capita diagram is that each consumer in the rich country reaches a higher utility level than the consumers of the poor country. This is a result of the definition of a rich country, which is that it has more capital per labour unit than poor countries.

The illustration points out the fact that the production possibility set (PPS) for the poor country has a steeper slope, in general, than the richer country’s PPS. The reason for this is the higher capital/worker ratio in the rich country. The way the PPS’s are drawn indicates that the rich country has more capital in both absolute and relative terms than the poor country, while the poor country has more labour. This is not very unrealistic, as we know that most people in the world today live in developing countries while most of the capital (both financial and physical) is situated in the OECD.

Due to homothetic and similar preference paths in the two countries, we can draw the indifference curves in this diagram and let them represent the preferences for consumers in both countries. It follows from the figure that the relative price level in autarky for the poor country is lower than the relative price level in the rich country. This is due to the difference in relative factor abundance.

The curves illustrate that:

(28) \[-\left(\frac{P_s}{P_c}\right)_D^A > \left(\frac{P_s}{P_c}\right)_O^A \iff \left(\frac{P_s}{P_c}\right)_D^A < \left(\frac{P_s}{P_c}\right)_O^A\]

We know that none of the countries’ post-trade production is fully specialised and we can find the world market relative prices as we did in the last chapter, by drawing the world’s PPS and the representative consumer’s indifference curves. As already mentioned, the post trade world market price will then lie between the two autarky price levels.

The gains from trade stem from the fact that the consumption possibilities are no longer restricted by the PPS, but of the post-trade relative price line. When the new world market price line is drawn in the figure, as in diagram 7, we see that the utility levels in each country are higher than in autarky.
We see from the consumption point for each of the two countries that the consumers reach a higher utility level after trade compared to the pre-trade situation. The gains from trade is due to the fact that in the post-trade situation consumers can choose a consumption point outside the PPS of their own country.

The reason why the consumption point is found as in diagram 9 is given in the following. The world market prices are found from the knowledge of both the world’s production
possibilities and the world’s consumption preferences. World market prices are decided so
that:

(30)  \[ P_C \cdot C_C + P_S \cdot S_P = P_C \cdot C_C + P_S \cdot S_C \]

(31)  \[ P_C \cdot (C_C^D + C_C^O) + P_S \cdot (S_P^D + S_P^O) = P_C \cdot (C_C^D + C_C^O) + P_S \cdot (S_P^D + S_P^O) \]

Where post trade world market prices have top-print * and footprints denote respectively
consumption (C) and production (P).

If we assume that trade is balanced between the two countries, it will also be true that:

(32)  \[ P_C \cdot C_C^i + P_S \cdot S_P^i = P_C \cdot C_C^i + P_S \cdot S_C^i \]

Where \( i = D, O \)

The left side expresses country i’s total income. Let \( (C_C^i)^* \) and \( (S_P^i)^* \) denote i’s
production of the two commodities in the after trade equilibrium. The income, \( \bar{I} \), will then
be expressed as follows:

(33)  \[ P_C \cdot (C_C^i)^* + P_S \cdot (S_P^i)^* = \bar{I} \]

\( S \) and \( C \) with top-print * are the production levels in the point on the PPS tangent to the
post-trade relative price line, and \( \bar{I} \) is the resulting income level for the country in
consideration. The income level in each country depends on the country’s endowment of
production factors, the consumers’ preferences, and the other country’s factor endowment.

A country exposed to international trade has a consumption possibility set (CPS) outside
its own PPS. In opposite to the autarky situation, the consumption constraint is now given by
the income defined above and not entirely by own production. The income level constrains
the consumption and defines a new CPS.

The value of the consumption has to equal the income:

(34)  \[ \bar{I} = P_S \cdot S_C^i + P_C \cdot C_C^i \]

Implicit derivation with respect to \( C_C^i \) gives:
\[ 0 = P_S \frac{dS_C^i}{dC_C^i} + P_C^* \]

This gives us the slope of the CPS in the diagram 8:

\[ \frac{dS_C^i}{dC_C^i} = -\frac{P_C^*}{P_S^*} \]

We see from this that the CPS has a slope equal to the relative price line. We then only need one possible consumption point to identify the CPS in the diagram. The production point obviously has to be a possible point for consumption. The consumption constraint is then given by the line through the equilibrium production point with slope of the negative of relative world market prices.

The PPS tangent that has a slope equal to the relative price on the world market then defines the consumption constraint. The optimal consumption point will then be the point where one of the indifference curves is tangent to this CPS.

### 3.2.2 The S sector produces non-traded goods

In this section the discussion continues within the HOS-framework, but one of the commodities is assumed to be non-traded, as it was in Mundell (1957). The assumption is that the non-traded goods are S goods, and they are interpreted as services.

The other sector can be interpreted as, for example, manufacturing. For simplicity, services are assumed to be totally immobile between countries, while manufactures can be traded without costs. These assumptions can be proved to be wrong; in reality some services can be traded and some manufactures are costly to get from one country to another. These assumptions might, however, have some root in reality. Services are often produced at the consumption point. Health and care services, for example, are not possible to send from one country to another without moving either workers towards the consumption point or the consumers towards the production point. Manufactures, by contrast, can most often be produced in one country and bought by consumers in another country.
Obviously, times have changed in this respect with the remarkable improvements in information technology during the last decades. It is now possible to make use of the Internet to send information services from one country to another. In fact, it might be claimed that services are less costly to transfer from one country to another than manufactured products. This point is ignored here, but will be returned to in chapter 4.

An argument for assuming that services are less tradable than commodities, is that they may differ from country to country. Services might have more specific national characteristics than manufactured goods. Health and care services, for example, are easier consumed when both producers and consumers have the same cultural background, language, and habits. Information services, although transported at low cost, might be easiest to produce in the country where it is to be consumed, where producers have the knowledge of the preferences and needs of consumers and the know-how on infrastructure. The discussion of the non-traded feature of the service sector is continued in chapter 4.

From the literature in the HOS tradition (e.g. Mundell 1957) it is said that factor mobility and free trade in the commodity markets are substitutes. If both capital and labour were mobile between countries, there would be no need for trade in C and S commodities. Then each country’s production point could be at the consumption points shown in diagram 10, without any trade present. The price and utility level would then equal the free trade equilibrium even in the absence of trade in the goods market. If only one of the factors of production is mobile, however, trade in one of the commodities also have to be present to reach this point.

Let us now study the two different situations in turn, both in which the S sector is non-traded. The first situation with mobile labour is technically similar to the illustrated example in Mundell (1957). The other situation discusses a situation with mobile capital, and the illustration and discussion of whether or not the factor mobility is a substitute for trade in S-commodities will be somewhat different from the first case. The framework here is static, which means that we assume that there is no time lag from the movement of labour or capital investment takes place till the actual production appears.

3.2.2.1 Labour mobility, free migration

Let us assume that the OECD countries accept free immigration, and that the labour force in the third world is totally rootless, moving wherever the wage is higher. Further, let us
assume for now that the labour moving to the OECD still belongs to the third world in the sense that their payment is returned to the poor country, and the labour sited in the OECD still consumes in the third world countries.

The flow of labour from the poor to the rich country will cause a shift in the PPS in both countries. The PPS of the rich country moves northeast in the diagram. The shift creates a larger vertical than horizontal distance along the axes to the ‘old’ PPS curve. The reason for this is that the S industry is relatively more labour intensive than the C industry. The opposite is true for the poor country. The PPS curve moves southwest in the diagram, and the negative shift is also larger in the vertical than in the horizontal direction.

The slope of the two countries’ PPSs is getting more similar; the rich country gets a better chance to produce S commodities, and the poor country is paid for the ‘labour-loan’ in C products. The countries get the same output and consumption as in the free trade equilibrium. Diagram 11 shows the rich country’s PPS and the amount of C goods that they send to the poor country to repay the labour loan.

**Diagram 10 Production and consumption in OECD after labour immigration**

![Diagram 10](image)

The new PPS drawn in diagram 11 lies northeast of the ‘old’ PPS. The new labour that comes to the country expands the production possibilities. The expansion is larger for the S goods than the C goods due to the fact that S goods uses labour more intensively in production. As long as the migrating labour is assumed to consume in its home
We know that since the S sector is non-traded, the consumption of S goods must equal the production of S goods in each of the two countries.

We now have the consumption possibility set defined as:

\( P_c^* (C_C^{'})^* + P_S^* (S_C^{'})^* = \bar{I} \)

\[ \Rightarrow \]

\( (C_C^{'})^* = \bar{I} - \frac{P_S^*}{P_C^*} (S_C^{'})^* \)

\( \bar{I} \) is the income of the country defined from production, own endowment is given by the PPS before the labour movement, and world market prices in world equilibrium. \( S_p^{' \prime} \) is the production of S goods in equilibrium. The consumption of C goods is found by finding the tangent to the PPS before the capital movement that has a slope equal to the relative prices in equilibrium after the labour movement, and find the point on this tangent where S equals the produced level of S in the economy.

\( \Delta C \) has to return to the poor country. The reason for this is that the labour should be paid, and the consumption point of the OECD must lie on the curve representing the income from trade with slope \( \frac{P_C^*}{P_S^*} \).

The labour force is far from mobile between poor and rich countries. This is due to political regulations in the OECD, as well as to travelling costs. In addition, people sometimes seem to be of an immobile nature; most people are not willing to leave their home countries.

There is also a question to be asked about the homogeneity of labour in different countries. Differences in culture, language, schooling and knowledge are some of the aspects that might make the labour force different. I will return to a discussion of these and related aspects in chapter 4.
3.2.2.2 Capital mobility

The next situation to study is a world where capital is perfectly mobile between countries, while labour is immobile. This may be a situation closer to the real world, especially if we consider financial capital to be the factor of production. Trying to illustrate the capital flow in the same way as done in last section, the model would predict a flow of capital from the rich country to the poor, and a repayment in S goods.

Diagram 11 Capital movement from OECD to D, PPS for the developing countries

Diagram 11 shows the same effect of capital mobility that was shown for labour mobility in diagram 10. The assumption here must be that S-goods are tradable, so that the capital owners in the OECD can get their payment for the

\( \Delta S \) should be sent as repayment to the rich country. The S goods are immobile between countries, though, and the repayment is impossible. The situation described in diagram 11,
will not occur. This is a point not mentioned in Mundell’s article. He argues that the two are substitutes, and show this by one example. Steel is then not freely traded, but capital is. Steel is relatively capital intensive in production, and his result is then the same as the first case shown here, and illustrated in diagram 11.

The prediction from this model is, however, that the capital would flow into the poor country till the poor country was more capital abundant than the rich country, and send capital intensive goods back as repayment.¹²

This can be illustrated in an edgeworth box, as done in diagram 12.

Diagram 12 Edgeworth box with capital mobility and a non-traded good

Diagram 12 illustrates an example of an initial situation at point E where the rich country is endowed with relatively more capital than the poor country. The labour intensive good is assumed to be non-traded, while the capital intensive good can be traded without cost. The capital will then move to the poor country, and in an amount such that the rich country becomes less capital abundant than the poor country. The capital movement is illustrated with the blue vector from E to P, and P is the production point. The repayment to the rich

¹² This argumentation can be found in Venables and Norman (1995)
country for the capital loan comes in capital intensive goods, illustrated by the green vector from P to C. The slope of this vector illustrates the factor content of production of C-goods. The preference direction for the poor country is northeast in the box, while the preference direction for the rich country is southwest. The indifference curves for both countries are convex. In the consumption point, C, both economies have reached a higher utility level than in E.

Rosenstein-Rodan (1943) expressed capital movements to one region as industrialisation of that same region (i.e. the reference in the beginning of this chapter). Using this terminology, the above expressed technical situation will lead to industrialisation of the poor country in the world and a deindustrialisation of the OECD countries. For certain, these large movements of capital are not observed. So the paradox regarding capital movements might not only be that poor countries are not industrialised, but also that the industrial countries are not deindustrialised.

A reformulation of the Lucas paradox might in this case be necessary:

>If the labour intensive goods are less tradable than the capital intensive goods, capital is mobile and labour is immobile between countries, the prediction from this model should be that capital moves to the developing countries to the extent that as a result the poor countries have relatively more capital than the rich countries. These flows of capital are not observed.

If one accepts that deindustrialisation of the industrialised countries will not happen, because of political-, technical- or other constraints, one have to argue outside the model. But if such a constraint is apparent, the model, and figure 12, shows that the possibility that labour intensive goods tend to be less tradable than capital intensive goods should be added to the discussion of the Lucas’ paradox.
4. Two small countries, one rich and one poor

The situation considered in this chapter, is one with two small countries, one that is rich and one that is poor. As shown by equation (39) the world market is assumed to be more capital scarce than the rich country, but more capital abundant than the poor country.

\[
\left( \frac{K}{L} \right)^P < \left( \frac{K}{L} \right)^W < \left( \frac{K}{L} \right)^R
\]

\(P\) indicates the poor country, \(W\) indicates the world market, and \(R\) indicates the rich country.

According to the neoclassical production function this will lead to the highest rent on capital in the poor country, the intermediate rate at the world market, and the return on capital will be smallest in the rich country.

The situation regarded in this chapter is similar to the one in the previous section in many ways: There are two sectors of production, \(S\) and \(C\), there are two factors of production, \(L\) and \(K\) and, finally, the production functions have the neoclassical properties. The initial situation is that \(C\) goods and capital are mobile between the countries, while \(S\) goods and labour are immobile. According to the discussion in the last chapter, trade between the rich and the poor country will not occur in the initial situation. Trade might occur, however, if either labour or \(S\) goods were mobile between the two countries.

A new source of income for the rich country is introduced, for which no inputs are used in production. This resource rent can be used to buy traded goods at the world market or it can be invested in the poor or the rich country and transformed into physical capital.

The rich country faces a demographic situation similar to the one described for the OECD countries in chapter 1. The government has, therefore, decided to save and invest some of the present income from the natural resource, for future consumption needs. Norway is the rich country example, and the investment decision will therefore be regarded as the Norwegian Government’s decision on how to invest the Norwegian Petroleum Fund (PF).
The world market interpretation will be the OECD countries, or more precise, similar to the benchmark portfolio of the PF today. The portfolio that the PF is invested in today will thus be called a world market portfolio investment.

The government has four different possibilities:

- Invest in own country
- Invest in a world market portfolio
- Invest in the poor country
- Open up for free migration

There are two important questions to be asked before the investment decision is made, both of them mentioned in the introduction.

The first is the main question that appears to be important in each of the alternative possibilities above:

*Can the PF be transformed into goods that fit the future consumption needs of an ageing population?*

The second important question that applies to investment possibility one and two respectively, is whether the coming transformation of the pension schemes in the OECD-countries will lead to lower rents on capital in this area.

There is now assumed a time lag from the investments or factor movements take place till the effect on production occurs. Two time periods will be regarded; the first is when the decision is made and either the investment or the migration takes place, while the other is when the effect on production or import occurs.

In the following, the four different investment possibilities will be discussed in turn.

### 4.1 Invest in own country

If the fund had been invested in own country, the PPS for period two would shift out and lie northeast in the diagram compared to the PPS without the investment.\(^\text{13}\) The intersection

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\(^\text{13}\) The PPS in period two without effects from the fund is assumed to equal to the period one PPS throughout this chapter. This is only for simplicity, and the challenge facing the period two consumers may than be regarded as there are more of them to share the same production as before.
with both axes shift due to the fact that capital is a factor of production in both the S and the C sector. The relative shift along the S axes is, however, smaller than the relative shift along the C axes due to the fact that C goods uses capital more intensively in production.

Diagram 13 Invest in own country
If the shift in the PPS had been larger northwest than northeast in the diagram, the utility would increase more than shown in diagram 13. Another way to say this is that if the petroleum income could be transformed into labour units in period two, the utility increase from the petroleum income would be larger.

When capital is invested in the most capital intensive country it is also invested in the country where the rent on capital is lowest, according to the neoclassical production function. So, one would believe, that if other investment possibilities are present, it would be profitable to choose one of the other possibilities. The logic behind this is that the rich country already produces C goods in a large amount compared to the poor country and the world market.

Further the wage is higher the larger the rate of capital to labour in the economy. In Norway it has been discussed during the 90s that the wage rate is high, and perhaps to high, due to the income from the petroleum sector. Other countries have after a period of high incomes from natural resources experienced a decline in the competitiveness in other industries, and with the disappearance of this income, these countries’ income has declined.

The investment in own country is, according to the neoclassical model, more profitable the less capital abundant the country is. We see from the figure that the production possibilities increase relatively more for the capital intensive good than for the less capital intensive good. Then it will also be the case that the more consumers tend to prefer C goods to S goods, the more profitable the investment.

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dramatically. This is called the *Dutch disease argument*, named after the situation in Holland during the 1980s and -90s after large incomes from the gas sector were put into the economy and the flow of income disappeared.\(^{14}\) If the capital were invested in Norway, the wages would increase even further. As already mentioned in chapter 2, it is said in the regulation of the PF that the total fund has to be invested abroad. The argument for this is to avoid further pressure in the economy.\(^{15}\)

(...) *the need to maintain and protect the fiscal budget as a management tool, the need for a diversified industry structure, i.e. avoid “Dutch disease”, and the need to stabilize the Norwegian economy by avoiding pressure on the exchange rate and decrease in interest rates that might have happened if the fund was invested in domestic markets.* (Ministry of Finance, 1990)

In the next session we will return to a discussion of the world market portfolio investment, the investment option chosen today.

### 4.2 Invest in a world market portfolio

The PF is today, as discussed in chapter 2, invested in a world market portfolio. It is therefore important to ask whether a world market portfolio investment can be transformed into consumption needed in the following periods. Second, the expected return on the world market portfolio, in a period when many countries are expected to build up similar funds, has to be discussed.

The PPS for period two describes, as before, the production possibilities when all capital and labour in the economy is fully utilized. The capital in the economy is the amount of capital the economy has without using any of the saved money on capital equipment. This might be assumed to be the same capital that the country had in period one, i.e. the domestic savings was equal to the capital depreciation in period one.

When the savings from period two have been invested in a world market portfolio, the PPS will be unaffected. The Consumption possibility set (CPS) though, will not be given by


\(^{15}\) Author’s translation from Norwegian
the PPS, but have an overall horizontal distance from the PPS equal to the fund’s value in terms of traded goods. The fund’s value will equal the sum of savings from period one and capital rents given at the world market.

**Diagram 14 World market portfolio investment**

Diagram 14 shows the second period PPS and consumption possibilities for the rich country if investment in period one is done at the world market, and put into the economy in period two. The horizontal distance from the PPS and the consumption possibilities are always equal to the value of the fund and its returns accounted in C-goods. The intersection with the second axes is the same for the PPS and CPS. This is due to the fact that the fund and its returns can only be used at the world market, and the only tradable good there is C-goods. If one tried to buy S-goods for the fund and its returns, the only effect, except the distributional, would be inflation. If a certain group for example pensioners received the fund and its returns, they would only exclude other consumers. The reason for this is that all resources are fully utilised in this economy.

The intersection with the vertical axes is the same for the PPS and CPS. This is due to the fact that the fund and its returns can only be used at the world market, and the only tradable good is C-goods. If one tried to buy S-goods for the fund and its returns, the only effect, except the distributional, would be inflation. If a certain group, for example pensioners,

---

16 The analysis here has some similarities to Holden (1999), where structural effects from petroleum income and Dutch Disease arguments are discussed.

17 It will be consumed more S goods after the fund is put into the economy, because the structural change indicates that less C goods and more S goods are produced in the economy.
received the fund and its returns, and wanted to buy S goods only, they would exclude other S goods consumers. The reason for this is that all resources are fully utilised in this economy.

The yellow arrow indicates the reduction in own production of C goods in period two, while the blue arrow indicates the increased consumption of C goods in period two. The pink arrow indicates the increased production and consumption of S goods in period two.

The result from the investment at the world market will not only result in a change in the consumption in period two, but also a change in production: Less traded goods and more services will be produced in Norway. The Dutch disease argument will then also be present here: A structural change in production involving a change towards production of labour intensive goods will lead to what can be called “a higher scarcity in the already scarce factor”, labour. The wage will then rise and the competitiveness in other industries than petroleum thus decreases. If the petroleum income disappears in later periods, the country will experience a decrease in income, and consumption of traded goods.

The diagram shows that the portfolio investments will give a higher utility in period two than the case with no saving. We see that the country’s production structure changes, the manufacturing production decreases, while the service production increase. Consumption of both goods increases.

As is seen from the figure, the more the preferences tend to choose services than traded goods, the less is probably the utility increase from putting the fund into the economy in period two, for example in the form of pension payments.

Norway is one of few countries that have tried to calculate the amount of traded goods relative to non-traded goods in consumption. In 1989 75% of GDP was in non-traded goods while the rest was traded goods (Norman, 1993).

It is reasonable to assume that the elderly part of the population consumes services like health care and social care to a larger extent than the rest of the population. When the rate of elderly to the younger part of the population increases, it will then also be reasonable to assume that the preferences of the representative consumer will move towards non-traded goods. This discussion goes outside the model used here, where homothetic preferences is

---

18 Crucial to this argument is the two-period framework in use here. If the time horizon was unlimited, and the fund was set aside to give the country a permanent flow of income for the future, the wage increase would then be much smaller and be permanent and therefore cause no problem.
19 The utility concept is not cardinal as discussed before, so it is actually not possible to compare changes in utility in different preference regimes.
20 In 1995 the National Accounting procedure was changed and the country no longer gives statistics of this. Rødseth (2000) explains that the distinction between traded and non-traded goods is not evident.
assumed, but this tendency will probably lead to a smaller shift in the utility curve, which will be important in comparison with the other alternatives.

The answer to the first question regarding whether or not the PF investment will make the consumption needed in period two available, will then be:

The more preferences tend to prefer S goods to C goods; the utility increase from increased consumption possibilities for C goods will be less than utility increase from increased consumption possibilities for S goods.

The second question about whether or not the expected change in pension schemes for most OECD countries from paygo to funded schemes, will result in the returns on the fund being smaller than expected, i.e. the horizontal distance from the PPS to the CPS in diagram 14 being smaller than expected.

The answer from the theory is clear; if funding increases savings at the world market, the rent on capital will decrease, and the horizontal distance between the PPS and the CPS will decrease as the savings at the world market increase.

The negative returns for 2001 and 2002 for the PF stem from a far too short period to say that it is a result of decreasing returns on capital. The argument that this is only an outcome of business cycles seems obvious.  

However, some, i.e. the famous speculator George Soros, argues that we face a period of depression in the Western economies and that it is time to focus on other potential growth engines. He argues that these growth engines could be the third world countries (Soros, 2001). What supports his view within the neoclassical models is the prediction that increased savings in the OECD, or increased funding as long as it leads to increased savings, leads to lower returns. It is nothing in the theory, however, that would predict negative returns.

4.3 Invest in the poor country

As shown in chapter 3, capital investments in the poor country may not happen in a situation where services are non-traded and labour is immobile between countries. In this

21 The PF, although giving negative returns the last years, has done better than the reference portfolio, meaning that the fund is managed well.
section a situation with partially non-traded services will be regarded. It is then possible to invest in transforming ‘pre-investment non-traded goods’ to ‘post-investment traded goods’.

First, let us discuss why the question about making initially non-traded goods traded, is worth asking. Diagram 15 shows how the possibility of buying S-goods from a poor country could place the CPS with a vertical instead of a horizontal distance to the PPS. With the preferences assumed, this would lead to a higher increase in utility in period two than the portfolio investment.

Diagram 15 If S-goods were tradable, and the PF used on services

The increase from utility level U1 to U2 is the increased wealth in period two from a portfolio investment while the increase from U1 to U4 is the increased utility from using the fund to buy services. Utility level U4 is a higher utility level than U2. The whole reason for

Diagram 15 is the same as 14 but a new red consumption possibility line is added. This red line has an overall vertical distance from the PPS equal to the fund and its returns accounted in S goods. This would be the new consumption possibility set if S goods, and only S goods, were tradable, and the fund was invested at the world market in period two. We see that with the assumed preferences, the utility from the world market investment is larger in this case than when C goods are tradable.
the former representing a larger welfare improvement is the preference structure that shows a
tendency towards S-goods. The opposite would be true if the consumers tended to prefer C-
goods.

This figure and the discussion so far have only expressed that it would have been better
for the rich country with an increased need for services in the future period if services had
been traded. So why is this called an investment in the poor country? The only possibility for
doing so is if there are barriers to trade in services that are possible to break down. In the
following such barriers are assumed. It is possible to remove the barriers at a cost.

The rationale for assuming that it is possible to transform a non-traded good, for example
services, to a traded one, can be of cultural, geographical, informational or institutional
character. The cultural character might be language. If health care for example should be
traded, many problems would arise, at least there would be a geographical barrier that might
have made the trade impossible or the cost extremely high. But if consumers of health
services were willing to move towards the production point, the barriers to do this would
probably be of cultural art. A common language would for example be needed. Further
norms, ethics and other differences might create barriers. The cost would then be to finance
language and other cultural information and training, and/or simply the transport cost for the
consumers that have to move to the production point. An example to illustrate this point is
that Norway has ‘imported’ health services through a state led initiative that sent patients out
of own country.22

The lack of willingness to invest in poor countries’ potential high return projects may also
be that the risks are considered too high. The cost could then be to create institutions, for
example banks and law institutions.

Two situations will be regarded in the following; the first assumes a trading cost of fixed
cost nature while the second consider a unit cost to trade. The unit cost can be reduced if
investment is done.

In reality the cost might consist of two elements, one unit cost and one fixed cost. For
simplicity, however, the two elements are here analysed separately.

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22 This has been called pasientbroen (bridge for patients). See for example Botten, G., Grepperud, S. & S.M.Nerland
(2003).
4.3.1 Fixed costs

If the assumed cost is of fixed cost nature, the question of whether the services are traded or not, is only a question of whether or not the cost is paid. If the fixed cost can be paid by capital alone, the result will be that the petroleum fund can be used to pay that cost and to buy services in the next period. The amount of services Norway gets in the second period is in this situation equal to the difference between the PF and the fixed cost, accounted in S goods.

The preferences, the size of the fixed cost, and the amount of savings will decide whether or not this will lead to a higher utility in period two than the case with portfolio investment.

Diagram 16 The effects in the rich country; S-goods mobility, fixed cost

Diagram 16 shows situations where S goods can be traded after a fixed cost has been paid. In diagram 16a utility increase from paying this fixed cost and buy S goods are higher than buying C goods with no costs. In figure 16b the opposite is true; it is better to buy c goods than paying the fixed cost and then buying S goods. The only difference between a and b is the size of the fixed cost.

The vertical distance from the PPS to the CPS is then equal to the difference between the return on the fund minus the fixed cost, counted in S goods.

Diagram 16a shows a situation where the utility from investing in the poor country, U3, is higher than the utility from the world market portfolio, U2. Diagram 16b illustrates the
The opposite situation where the utility from the investments in the poor country, U1, is lower than U2. The difference between the two situations stems from different size of the fixed cost only.

The effects in the poor country will be similar to the situation described in chapter one, but here they will actually get the capital investments because the repayment in S-goods can occur.

**Diagram 17 The effects in the poor country of S-goods mobility and a fixed cost**

Diagram 17 illustrates the effect in the poor country from the investment in that country. The new, brown, price line defines the CPS for the poor country. The poor country will gain from the capital and the transformation of services from non-traded to traded goods, as long as the price-line is getting less steep. If the fixed cost is as illustrated in diagram 16a, the new price line is as steep as the tangent in the point where U3 and the yellow line is tangent. The ΔS is the amount of services that goes to the rich country, as repayment of the capital loan.
So, if there is a fixed cost to make the services traded, the CPS’ intersection with the second axis will shift upwards as long as the savings increase the fixed cost. Whether the point where the indifference curve is tangent to the CPS is superior to the tangent point with a portfolio investment, will depend on:

- Preferences
- The rent on the portfolio
- The size of the fixed cost

The flatter the indifference curves, the higher the savings and the smaller the fixed costs the more likely it is that the direct investments leads to the highest wealth.

The arguments for modelling the costs as a fixed cost instead of a variable cost depending on the amount of S-goods that is traded, is not very robust. The best argument for doing so might be that it is easier to analyse, because the after trade (and after cost) relative price will be the same in both countries.

It might also be a problem from modelling the cost as consisting of capital alone. If the cost consists of teaching nurses or labour in the poor country Norwegian, the cost will probably not consist of capital alone, but also use Norwegian labour. Then the access to labour intensive goods, also have to be valued from the fact that labour, the scarce factor is used to get this access.

In the next session it is assumed that the cost is of unit cost type, and that the cost uses factors of production in the same amount, as does consumption of S goods. The investment to reduce this cost is however assumed to consist of capital alone. By allowing the investment to consist of capital alone, some generality and reality is lost, but the gain is the simplicity of the analysis.

4.3.2 Unit costs

In this session the PF as a whole is invested to reduce the unit cost on trade in S goods. The gains from this investment are the gains from trade in S commodities. The unit cost is modelled as what is called an iceberg trading costs. As the name indicates, these costs are modelled by assuming that some of the goods shipped *melt away* before they reach the consumers in other countries.
If the iceberg costs are so high in the beginning that no trade in services occur this must imply that:

\[
\frac{\frac{P_C}{P_S}}{\frac{P_C^p}{P^p_S}} > \frac{\frac{P_C}{P_S}}{\frac{P_C^r}{P^r_S}}
\]

but

\[
\frac{\frac{P_C}{P_S}}{\frac{P_C^p}{P^p_S}} < \frac{\frac{P_C}{P_S}}{\frac{P_C^r}{P^r_S(1-t)}}
\]

Where \( P_C^i \) indicates the consumer price of commodity \( k \) in country \( i \) and \( t \) is the proportion of good \( S \) that melt away during transportation. This indicates that the cost uses factors of production in the same amount as the consumption of \( S \) goods.

Let us assume that \( t \) is a decreasing function of the savings used for reducing it:

\[
t = t(I), \quad t'(I) < 0
\]

\( I \) is the investment to reduce the unit cost on trade in \( S \)-goods. The sign on the derivative indicates that if savings are invested to reduce the trading cost, then iceberg cost is actually reduced.

All three different signs on the second derivative are possible. The most common assumption to make is that the second derivative of an investment is the opposite of its first derivative, so that the effect of the investment decreases with the size of the same investment. The logic of assuming this is that there are gains to exploit by the investment, but when investment is made, there is less and less left to exploit.

It might be possible in this case, however, that the second derivative is negative, meaning that such an investment has an increasing effect on the iceberg trading cost. To understand this, we first have to look at what kinds of investments could be done to reduce unit trade costs on services. This will certainly depend on what kind of services we have in mind.

Health- and care services for example, are most often produced at the consumption point. The easiest way to make \( S \)-goods traded might be to transport the consumers to the production point in the poor country. The unit trading cost could then be interpreted solely
by transport cost of the pensioners or patients. Probably, though, there are more costs involved than that. If the elderly Norwegians are going to spend their everyday lives in another country than their home country, this might make a social disutility that could be compensated. Further and maybe more easily modelled, is the cost of teaching nurses and care keepers Norwegian or the pensioners English or another common language in their host country. In such inter-country relationship, an understanding of culture and tradition in the other country is important. Schooling in culture, history and language for both producers and consumers could thus make a cost.

Information services on the other hand, can be produced far away from the consumption and transported without cost or with small costs through Internet or by use of telephone for example. The cost of making these goods traded might then be to match producer and consumer, by gathering the information on consumers’ needs and producers’ resources.

Both education and knowledge gathering might have increasing returns features and positive externalities. The more nurses one have taught English, or Norwegian, the more will maybe nurses at the same hospitals without schooling, get to know. And know-how on consumers’ needs and producers’ resources might create better access to more such knowledge.

As the assumption here is that there are only three countries in the world, one big world market, one small poor country, and one small rich country, there is no need for assuming anything about the second derivative to secure that there will be a solution. The PF size is limited and so is the unit cost. Therefore there is a constraint on the investment that means that although increasing returns on the investment, it has a defined solution. Even if the assumption on the second derivative is not deciding whether a solution to the problem occurs or not, it is certainly crucial to how profitable the investment is. If the second derivative is negative the probability that the investment in the poor country is superior to the other investment possibilities is higher.

For trade to occur in this situation, one must assume that the price for the consumers in the rich countries will be lower for goods from the poor country, even after some goods melt away, than the autarky level.

---

23 If the world market consisted of many countries that had the same production possibilities as the poor country described here, one could, if the second derivative of the t function was positive, get the solution that it would be profitable for the rich country to invest in more than one poor country. But with the assumptions made here, all three possible signs securces

(i) \( t^{\prime\prime} < 0 \)

(ii) \( t^{\prime\prime} > 0 \)

(iii) \( t^{\prime\prime} = 0 \)
Whether or not it is preferable to the portfolio investment, to invest to reduce the unit trade cost, will depend on:

- The function \( t = t(I) \), where \( t' < 0 \).
- Preferences
- Rent from the portfolio investment

For the poor country the same is true as was true in the last chapter; as long as the price-line gets flatter, the consumers reach a higher utility level.

Using top-print \( R \) to denote the rich country and \( P \) to denote the poor country, and bottom-letter \( C \) to denote the traded good and \( S \) to denote the initial non-traded good, the relationship between consumer prices in the two countries, if trade and no specialization occur can be expressed by:

\[
\left( \frac{P^R_C}{P^R_S} \right) = \left( \frac{P^P_C}{P^P_S (1 + t(I))} \right)
\]

4.3.2.1 The Norman and Venables (1995) model

The Venables and Norman (1995) study introduces an iceberg trading cost, in one of the sectors in the HOS framework, in the way done here, and find solutions to when trade occurs and not, similar to equation (43). An introduction to their model is given in the following. They first assume that both goods are tradable, one with a unit iceberg trading cost and the other costless, while the factor of production are perfectly immobile between countries. The algebra and solutions to this state will be given in this section. Secondly, they open for the

\[\text{24 The notation has been changed to fit to the notation used in this thesis.}\]
possibility of both factor trade and goods trade; which will be the combination of the two proposals here in section 4.3 and 4.4. Section 4.4 analyse the situation where capital is mobile while labour is mobile only at a cost.

The difference regarding the goods trade with unit costs, as discussed here, and the Norman and Venables (1995) framework, is the planning perspective. The Venables and Norman (1995) is a technical solution to when trade actually occurs and when trade does not occur, without any strategy solution, as to whether or not an investment to reduce the trading cost should be done in the period before.

Venables and Norman (1995) set up a full model in the HOS-framework with iceberg unit trade costs in one sector, allowing for factor mobility. The framework here is therefore similar to appendix B, but an additional element, the iceberg trading cost, is incorporated. They allow for the possibility of factor mobility of a composite factor.

Venables and Norman (V&N) let the commodity that is traded with no cost be a numeraire. In the following C goods will therefore be taken as a numeraire, resulting in the price of C goods being normalised to one:

\[
P^C_C = P^R_R = 1
\]

The relative price will therefore be expressed as

\[
\frac{P^i_S}{P^i_C} = \frac{P^i_S}{1} = P^i_S \quad i = P, R
\]

They assume non-specialisation and perfect competitive economies in addition to identical technologies in both countries. The unit cost functions, \(b_1\) and \(b_2\) will then equal the prices:

\[
P^i_k = b_\lambda(w^i, r^i) \quad i = P, R \quad k = S, C
\]

Further the equilibrium factor prices can be expressed as a function of the price of good one:
w and r indicates the wage and the rent on capital as before. The sign of the derivatives result from the assumption that S is labour intensive in production, and expresses a Stolper-Samuelson effect.

Output levels are determined by factor market clearing for each factor in each country. By use of Shephard’s lemma this can be written as:

\[ \frac{\partial L_i}{\partial w} = \frac{\partial b_s(w^i, r^i)}{\partial w} + \frac{\partial b_c(w^i, r^i)}{\partial w} \]

\[ \frac{\partial K^i}{\partial r} = \frac{\partial b_s(w^i, r^i)}{\partial r} + \frac{\partial b_c(w^i, r^i)}{\partial r} \]

Where \( K^i \) and \( L^i \) indicates the exogenously given amounts of capital and labour in country i.

Norman and Venables define a variable, z, which expresses the relative factor endowments in the trading world (which consists of the rich and the poor country only) and the two countries, respectively:

\[ z^R = \frac{L^R}{K^R} \]
\[ z^P = \frac{L^P}{K^P} \]
\[ z^W = \frac{L^P + L^R}{K^P + K^R} \]

The world endowment ratio is a weighted sum of the two countries’ endowment ratios:

\[ \lambda z^R + (1 - \lambda) z^P = z^W \]

The weighting factor, \( \lambda \), is given by:

---

25 This same relation, although in relative terms, is expressed in the fourth quadrant in diagram B1.
Each country’s relative supply of goods, $X$, can be expressed as a function of goods prices and relative endowment ratios:

\[
\lambda \equiv \frac{K^r}{K^w}
\]

(54)

The signs of the derivatives are results of the assumed factor intensities, and they express Rybyszynski effects.

The demand side of their framework is expressed by use of an aggregate consumer in each country, with utility level $u^i$ and expenditure function per unit utility $e(P^i)$. The consumer’s budget constraint in autarky can then be expressed as:

\[
e(P^i)_u^i = w^i L^i + r^i K^i
\]

(56)

Due to homothetic preferences the ratio of consumption, $S_C/C_C$, can be expressed as a function of goods prices:

\[
\left(\frac{S^i}{C^i}\right)^i = D(P^i) , \quad \frac{\partial D(P^i)}{\partial P^i} < 0
\]

(57)

The autarky equilibrium is then found by finding the relative quantities and the relative price levels from (55) and (57). Factor prices are then found from (48) and (49).

They assume for simplicity, as done above, that transactions use primary factors in the same proportion, as does consumption. The transaction costs of country $i$ consumers can then be expressed by:

---

26 This could be illustrated in the way done in the first quadrant in diagram B1
27 This could be illustrated by the fourth quadrant in diagram B1.
Where \( t \) is the same as the iceberg trade cost named \( t \) earlier in this section.

When the planning aspect is introduced within this framework, the \( \tau \) should be expressed as a function of the investments done to reduce it:

\[
(58') \quad e(P_S^I)\tau = e(P_S^i)\tau(I) = t(I) \quad \tau'(I), t'(I) < 0
\]

Norman and Venables assume that the costs are incurred in the source country, so that the function \( e(P_S^i) \) is evaluated at the source country price.

International trade does not now equate goods prices, but places a boundary on the difference between them. This boundary is as follows:

\[
(59) \quad e(P_S^R)\tau(I) \geq P_S^P - P_S^R \geq -e(P_S^P)\tau(I)
\]

In particular, when \( S \) goods is exported from the poor country to the rich country:

\[
(60) \quad P_S^R = P_S^P + e(P_S^P)\tau(I)
\]

Equation (60) expresses the same as equation (44).

By assumption the Poor country is relatively labour abundant, so we have:

\[
(61) \quad z^R > z^P \quad \Leftrightarrow \quad P_S^R > P_S^P
\]

If \( e(P_S^P)\tau(I) \) is greater than the difference between the prices, no trade will occur.

If \( e(P_S^P)\tau(I) \) is less than the price difference, it will be profitable to trade.

Whether or not trade occurs depends on the value of the function \( \tau \), on preferences, technology and endowments.
If \( z^w \) and \( \lambda \) are given, we can find four equations that have to be satisfied for the economies to be on the boundary between trade and no trade:

\[
\begin{align*}
(62) \quad X(P^R_S, z^R) &= D(P^R_S) \\
(63) \quad X(P^P_S, z^P) &= D(P^P_S) \\
(64) \quad P^P_S &= P^R_S + e(P^R_S)\tau(I) \\
(65) \quad \lambda z^R + (1-\lambda)z^P &= z^w
\end{align*}
\]

By varying \( \lambda \) we can trace the locus of this boundary in factor endowment space. The boundary values is called \( z^R \hat{=} \) and \( z^P \hat{=} \) and can be named the goods trade boundary.

By inverting (62) and (63) we get:

\[
(66) \quad P^R_S = \psi(z^R) \quad , \quad P^P_S = \psi(z^P)
\]

The function \( \psi \) gives the autarky price ratio as a function of the endowment ratio. With the assumed factor intensities it must be the case that \( \psi' \leq 0 \). The assumption here is that \( \psi' < 0 \).

Using equation (66) in equation (64) gives:

\[
(67) \quad \psi(z^R) + e[\psi(z^R)]\tau = \psi(z^P)
\]

Since \( \psi' < 0 \) it follows that \( z^R > z^P \), and from (65) we see that \( z^R \rightarrow z^w \) as \( \lambda \rightarrow 1 \) and \( z^R \rightarrow z^w \) as \( \lambda \rightarrow 0 \). Totally differentiating equation (65) and (67) and eliminating \( d\hat{z}^P \) give:

\[
(68) \quad \left[ \psi'(z^R)(1 + e'\tau) + \psi'(z^P) \frac{\lambda}{1-\lambda} \right] d\hat{z}^R + \psi'(z^P) \left( \frac{\hat{z}^R - \hat{z}^P}{1-\lambda} \right) d\lambda = 0
\]

Since \( z^R > z^P \) this implies:

\[
(69) \quad \frac{\partial \hat{z}^R}{\partial \lambda} < 0 \quad \text{and} \quad \frac{\partial \hat{z}}{\partial \lambda} > 0
\]
\( \hat{z}^R \) decreases monotonically to \( z^w \) as \( \lambda \) increases to unity. By varying \( \lambda \) from zero to unity it is possible to draw the trade boundary in an endowment box.

**Diagram 18 The goods trade boundary**

The curved line shows the boundary between trade and no trade, while the shaded area is the area where no trade occurs. When the investment increases, the boundary will shift northwest, with the result that it is less probably that the two countries end in a situation with no trade.

Since neither goods prices nor factor prices will be equalised in equilibrium, statements about factor content of consumption is not straightforward. Venables and Norman assume that technological coefficients are fixed and equal in both countries. With this assumption it must be the case that the factor content of consumption lays on the curved line. This is due to the fact that both (64) and (65) hold in equilibrium.

Equation (62) and (63) can be interpreted as implicitly defining the factor content of consumption in each country, since, with the same techniques of production in use in both countries, relative demands \( D(P^R) \) and \( D(P^P) \) generate factor demands \( \hat{z}^R \) and \( \hat{z}^P \).
If the initial endowment is at a point such as E, the factor content of consumption will be at a point such as Co. The factor content of trade, with given technologies used in production, could be: P exports vector ED of factors embodied in good S, and imports vector DC embodied in good C.28

### 4.4 Immigration

In this section we will return to a situation where S goods is non-traded by assumption. It is assumed, however, that the labour is mobile, but at a cost. First the fixed cost nature on labour movements is assumed, later the unit cost possibility is assumed.

An important discussion relating the mobility of labour is whether or not the labour in different countries is homogenous. The cost relating labour movements might consist of making different labour homogenous. Nurses from the poor country might have to learn

---

28 The gradients of ED and DC give the factor proportions used in production of goods S and C respectively, and the position of Co relative to E comes from the budget constraint, depending on factor prices and trade costs.
Norwegian before moving to Norway, to work at Norwegian hospitals. The assumption in the HOS framework is that the labour is homogenous within and between countries.\textsuperscript{29}

A second concern related to the mobility of labour is the distribution of income in the rich country when open up for migration. It is already shown within the simple neoclassical production function that with increased amount of labour to capital the ratio of wage to return on capital is reduced. This can also be seen within the HOS framework, as long as there is not perfectly free trade in commodity or factor markets.\textsuperscript{30}

Immigration of labour from poor countries to the OECD does not appear in large scale today. The reasons for this may be twofold; first the OECD countries do not want immigrants to come, second the poor country labour might want to stay in own country. Today the first is more realistic because we know that poor country labour wants to get into the OECD countries. However we do not know till what extent the poor country inhabitants are mobile. Labour might occur to be less mobile than one could expect by studying economic frameworks, for example like the one used here.

The OECD countries may have many different reasons to resist labour migration. Two examples are the following:

- Distributional concerns, as mentioned above
- Valuing own labour and migrating labour as heterogenous

The cost of immigration could then be the disutility from these negative effects of immigration. The investments to reduce it, or the fixed cost to remove it, could then consist of reducing or removing these effects. This could be done for example by:

- Implementing distribution programmes
- ‘Making the migrating labour and own labour homogenous’

The second point about making foreign labour homogenous to Norwegian labour could perhaps be done by schooling. This is similar to the discussion in section 4.3. Teaching foreign labour Norwegian languages and habits and so on could be done. On the other hand,

\textsuperscript{29} It is possible, though, that the problem of heterogenous labour could be analysed within a Ricardo-Viner framework, where the most usual assumption is sector-specific capital

\textsuperscript{30} This can be shown by for example shifting the curves in the fourth quadrant in diagram B1
the consumers in Norway could of course also learn about habits and cultural characteristics in the poor country.

The cost of making the immobile labour mobile will in the following be modelled first as a fixed cost and then as a unit cost on migration. As in the last section, the argument for assuming that the cost is of fixed cost nature is not robust, and perhaps the cost should be considered consisting of two elements; one fixed cost and one unit cost. For simplicity and illustration the two elements is analysed separately, as was also the case for section 4.3.

4.4.1 Fixed cost

If there is a fixed cost that has to be paid for the Norwegian Government to get access to the poor country’s labour, and the labour movement actually occurs, the PPS for Norway will shift out. The simplest possible case to analyse is the case where the fixed cost can be paid by capital alone, the repayment for the labour loan is in capital, and the migrants still consume in their home country. The Norwegian PPS will shift as shown in diagram 20.

**Diagram 20 Labour migration at a fixed cost**

The blue PPS illustrates one possible outcome of labour mobility at a fixed cost. The resulting utility level U is here drawn as a higher utility level than the utility level labelled
U2, when investment in own country is done. This will certainly depend on how high the fixed cost is, and it is indeed possible that the utility level when following this strategy is lower than the utility level when investing in own country. The higher the fixed cost, the higher is the probability that the other investment possibilities are preferable to the poor country investment.

4.4.2 Variable iceberg cost

The analysis of the unit cost on migration is following the Norman and Venables (1995) model. When allowing for factor mobility Norman and Venables take a general view and allow for movements of a composite factor. The composite is exogenously specified. It could be the case that labour migrates with some capital, or foreign investments might cause labour to move. The share of labour in the composite is given by a parameter $\alpha$. The movement of the composite factor is possible at a cost, $e(P^I_s)\gamma$.

Incorporating the planning perspective also in this section, the cost function in period two can be expressed by the savings done to reduce it, I, in period one:

$$
(70) \quad e(P^I_s)\gamma = e(P^I_s)\gamma(I)
$$

$$
(71) \quad \gamma'(I) < 0
$$

Further they allow for the possibility that factors moving can consume either in their destination country or in their country of origin. Migrated labour would probably not send all their earnings to their family and friends in their country of origin. It is not unreasonable, though, that some of their earnings would be sent back. Norman and Venables regard the two border cases, only; either the factors consume in their destination country or in their country of origin. The parameter $\beta$ takes the value 1 if all consumption takes place in the country of origin, while $\beta$ equals 0 if consumption takes place in the destination country.

The return to the composite factor can then be expressed as a function, $y$, of $\alpha$, $\beta$, r, w and $e$:

$$
(72) \quad y(P^I_s; \alpha, \beta) = \frac{\alpha w(P^I_s) + (1 - \alpha) r(P^I_s)}{\beta + (1 - \beta) e(P^I_s)}
$$
Where \( r, w \) and \( e \) are given the interpretations as in section 4.3.

Labour movement is discussed in this section, so the interesting case is when \( \alpha \) equals one. Equation (72) will then equal:

\[
(72') \quad y(P_s^i; 1,1) = w(P_s^i)
\]

when \( \beta \) equals one, and the migrating labour still consumes in their home, poor, country.

When \( \beta \) equals 0, however, and the consumption takes place in the rich, host, country, equation (72) will equal:

\[
(72'') \quad y(P_s^i; 1,0) = \frac{w(P_r^i)}{e(P_s^i)}
\]

The bounds that the possibility of factor movements place on the home and foreign values of \( y \) is given by:

\[
(73) \quad \frac{e(P_s^p)\gamma(I)}{\beta + (1 - \beta)e(P_s^p)} \geq y(P_s^p; \alpha, \beta) - y(P_s^r; \alpha, \beta) \geq -\frac{e(P_s^p)\gamma(I)}{\beta + (1 - \beta)e(P_s^p)}
\]

When the migrating labour consumes in the host country \((\beta = 0)\) this will equal:

\[
(74) \quad \gamma(I) \geq \frac{w(P_s^p)}{e(P_s^p)} - \frac{w(P_s^p)}{e(P_s^p)} \geq -\gamma(I)
\]

When the migration labour consumes in their country of origin \((\beta = 1)\), however, this will equal:

\[
(75) \quad e(P_s^p)\gamma(I) \geq w(P_s^p) - w(P_s^r) \geq -\gamma(I)e(P_s^p)
\]

We know that if labour movements occur it will be from \( P \) to \( R \), this will imply that the difference \( y(P_s^p; \alpha, \beta) - y(P_s^r; \alpha, \beta) \) will be positive in the initial situation. In this case the first equality/inequality will be the possible situations.
The following must then also be true (if trade occurs):

\[
\frac{e(P_S^R)y(I)}{\beta + (1 - \beta)e(P_S^R)} = y(P_S^P; \lambda, \alpha, \beta) - y(P_S^R; \alpha, \beta)
\]

When there is no trade in S goods it is possible to find the values of \( z^R \) and \( z^P \), respectively that gives the boundary for factor trade to occur. Calculating the following equations does this:

\[
X(P_S^R, z^R) = D(P_S^R)
\]
\[
X(P_S^P, z^P) = D(P_S^P)
\]
\[
\lambda z^R + (1 - \lambda)z^P = z^w
\]

The method is the same as in the former section, and it is possible to find a factor trade boundary, which is qualitatively similar to the goods trade boundary.

**Diagram 21 Factor trade and goods trade boundaries**
Norman and Venables consider both the possibility of factor movements and the possibility of goods movement at the same time, i.e. they allow the two situations described in section 4.3 and 4.4 simultaneously.

Whether goods trade or factor trade occurs, depend on the relative positions of the goods trade and factor trade boundaries. If the factor trade boundary is closest to the diagonal, the goods trade boundary is irrelevant. Since the factor trade boundary is closer to the diagonal, it has smaller international endowment ratio differences and hence smaller goods and factor price differences. Factor trade thus ensures that international differences in the price of labour and capital are never large enough to support a goods price difference as large as the transaction cost on S goods. The opposite will be true if the goods trade boundary is closer to the diagonal than the factor trade boundary. In the shaded area there will be no trade, neither factor nor goods movements.

In the area between the two curved lines there will be trade in either goods or factors depending on which boundary is the one closest to the diagonal. If the factor trade boundary is the one closest to the diagonal, there will be no trade in S goods. Whether or not there will be trade in C goods depends on the value of \( \beta \). If \( \beta \) is 1, the labour consumes in the country of origin and C is sent from P to R. If \( \beta \) equals zero however there will be no trade C.

The next discussion will then be under what circumstances the goods trade boundary would lie below the factor trade boundary. This is done formally in the Norman and Venables study.

Whether or not an investment to reduce the cost of migration is superior to the three other investment possibilities is thus dependent on:

- The size of the fixed cost or the unit cost function, \( \gamma(I) \), on migration
- The size of the fixed cost or the unit cost function, \( \tau(I) \), on trade in S goods
- Consumer preferences
- The return on the world market portfolio
- Production possibilities in own country
5. Concluding remarks

The following conclusions can be drawn from the analysis in this thesis:

- The demographic situation in the OECD countries results in a problem on how to meet future consumption needs. It is not obvious, though, that the solution to this problem is to change the existing pension systems.
- When saving for future consumption needs one should take into consideration that increased saving in one region might reduce the return on investments in that same region.
- One possible explanation to the Lucas’ paradox is that labour intensive goods tend to be more of the non-traded sort than the more capital intensive goods. Capital will thus not move to poor countries unless the rich countries are deindustrialised, and the poor countries industrialised.
- In addition to the expected return consideration, it is also important for a nation experiencing increased ratio elderly/working part of the population, to discuss which goods/services are needed in the future. Elderly needs services to a larger extent than the rest of the population, and when experiencing a shortage of labour supply, it should be discussed how the needed services can be put in place.
- Whether a world market portfolio investment is superior to investment in own country, investment in own country, investment in a poor country, or migration, depends on consumer preferences, the return on the portfolio investment, production possibilities in own country, trading costs on services, and the cost on migration. The higher the return on the portfolio investment, the more capital abundant the country is initially, the higher the trading costs on services and cost on migration, the more likely that the portfolio investment is the best investment strategy. Regarding consumer preferences one has to be careful about comparing different preference structure, but the more consumers prefer services to other goods, the more important is it that the investment strategy is able to obtain services. This could be an argument for the investment in the poor country or the migration strategy to be preferable to the portfolio investment.
References/Literature


Finansdepartementet (1990): Lov om Statens petroleumsfond, Act of June 22, 1990 (no 36, i Nr 10)


Appendix A

The SSB calculations

The predictions of the future labour force and the benefits needed to pay pensions are to a large degree dependent on the assumptions done in the calculus. The population numbers have been taken from the intermediate alternative from the last population projections done by the Bureau of Statistics.31 The assumptions made are:

- Immigration: 13 000 persons each year
- Fertility rate: 1.8
- Life expectancy: Grow at approximately 0.15 year per year for the whole period considered here
- The tendency to study: Approximately at the 2001 level
- Vocational occupational rehabilitation (attføring), disablement pension and AFP:32 Approximately at the 2001 level
- Retirement age: 67
- Participation rates and average working time ordered after gender, age, schooling, educational level, status as a retired, children etc: Approximately as in 2001
- Prices, wages and base amount:33 As in 2001

31 Statistisk sentralbyrå (SSB)
32 Avtalefestet pensjon (AFP) is a Norwegian term referring to a program for early retirement in Norway
33 Base amount (G) in Norway, was in 2002 (on average) 53 232 NOK. G is regulated according to changes in the average real wage in Norway and pension payments are counted in G’s.
Appendix B

A more mathematical description

B1 The factor market equilibrium

Assume that the production functions homogenous of degree one. The unit cost functions then depend on factor prices only, and not the scale of production. Let the unit cost function be given by:

\( c_i = c_i(w, r) \)

The letters label the same variables as in chapter 3. \( w \) is the wage and \( r \) is the rent on capital, \( i = S, C \).

The cost functions will then be expressed by:

\[
\begin{align*}
(B1) & \quad \frac{\partial c_i}{\partial w} S = c_{iS} S \\
(B2) & \quad X_S = c_S(w, r) S \\
(B3) & \quad X_C = c_C(w, r) C \\
&B4 \quad L_S = \frac{\partial c_S}{\partial w} S = c_{iS} S
\end{align*}
\]

By use of Shepherd’s lemma, the labour used in the S sector is found to be:

\[
\begin{align*}
(B4) & \quad L_S = \frac{\partial c_S}{\partial w} S = c_{iS} S \\
&B4 \quad L_C = \frac{\partial c_C}{\partial w} C = c_{iC} C
\end{align*}
\]

and similar for the C-industry. \( c_{ij} \) expresses the derivative of the unit cost function with respect to \( w \) and \( r \) respectively, \( j = S, C \).

We then have that

\[
\begin{align*}
(B5) & \quad c_{KS} S + c_{KC} C = K_0 \\
(B6) & \quad c_{LS} S + c_{LC} C = L_0
\end{align*}
\]

By differentiating the system:

\[
\begin{align*}
(B7) & \quad d(L_S) + c_{LS} dS + C(c_{LC} + c_{LC} dC = dL_0 = 0 \\
(B8) & \quad d(L_C) + c_{CS} dS + C(c_{KC} + c_{KC} dC = dK_0 = 0
\end{align*}
\]

By simplifying and finding the relative terms:

\[
\begin{align*}
(B9) & \quad c_{LS} S(\hat{c}_{LS} + \hat{S}) + c_{LC} C(\hat{c}_{LC} + \hat{C}) = dL_0
\end{align*}
\]

34 The calculus is the similar (although shorter) to the one done in Klette (2001) and Atkinson and Stiglitz (1980)

35 A relative change in a variable will be denoted in the following way: \( \hat{V} = \frac{dV}{V} \), \( V \) being any one variable.
\( \hat{L}_0 = \frac{dL_0}{L_0} = \lambda_{LS} (\hat{c}_{LS} + \hat{S}) + \lambda_{LC} (\hat{c}_{LC} + \hat{C}) \)

Where \( \lambda_{ij} \) gives the share of \( i = L,K \) that is employed in sector \( j = S,C \).

For the capital market, the corresponding expression will be:

\( \hat{K}_0 = \frac{dK_0}{K_0} = \lambda_{KS} (\hat{c}_{KS} + \hat{S}) + \lambda_{KC} (\hat{c}_{KC} + \hat{C}) \)

The expression \( \lambda_{ij} \hat{c}_{ij} \), \( j = S, C \) respectively, captures the changes in production levels, while factor substitution is captured by \( \lambda_{ij} \hat{c}_{ij} \).

The expression for \( \hat{c}_{ij} \):

\[ \frac{dc_{LS}}{c_{LS}} = \frac{(c_{LLS}dw + c_{LCS}dr)}{c_{LS}} \]

Where

\[ c_{LLS} = \frac{\partial^2 c}{\partial w^2} (w,r) \]
\[ c_{LCS} = \frac{\partial^2 c}{\partial w\partial r} (w,r) \]

\( c_{ij} \) is homogenous of degree zero in \( w \) and \( r \), due to the fact that \( c_{LS} = L_S / S \) only depends on factor prices.

We then have:

\[ c_{LLS} w + c_{LCS} r = 0 \]

\[ \Rightarrow \]

\[ c_{LCS} = -(w/r)c_{LLS} \]

By inserting (B16) in (B12):

\[ \frac{dc_{LS}}{c_{LS}} = \frac{(c_{LLS}dw - (w/r)c_{LLS}dr)}{c_{LS}} \]

We are interested in simplifying this equation and expressing it as a function of the technical rate of substitution. When assuming that the factor markets are characterised with perfect competition, the technical rate of substitution can be expressed in the following way:

\[ \sigma_S = -\frac{wc_{LLS}c_X}{rc_{LS}c_{KS}} \]

To express \( \hat{c}_{LS} \) as a function of \( \sigma_S \), (B18) can be rearranged in the following way:

\[ \hat{c}_{LS} = \frac{rc_{KS}}{c_S} \left( -\frac{wc_{LLS}c_S}{rc_{LS}c_{KS}} \right) (\hat{w} - \hat{r}) \]
The first expression on the right hand side of this equation will be called $\theta_{CS}$, and is the factor share of capital in the production of S goods. The next expression is the technical rate of substitution, $\sigma_S$.

This will give us the more simple equations:

(B20) $\hat{c}_{LS} = -\theta_{KS} \sigma_S (\hat{w} - \hat{r})$

(B21) $\hat{c}_{LC} = -\theta_{KC} \sigma_C (\hat{w} - \hat{r})$

(B22) $\hat{c}_{KS} = \theta_{LS} \sigma_S (\hat{w} - \hat{r})$

(B23) $\hat{c}_{KC} = \theta_{LC} \sigma_C (\hat{w} - \hat{r})$

If we insert these four equations in the factor market equilibrium, the following equation for the labour market will hold:

(B24) $L_0 = \lambda_{LS} \hat{S} + \lambda_{LC} \hat{C} - \theta_{KC} \sigma_C (\hat{w} - \hat{r}) \lambda_{LC} - \theta_{KS} \sigma_S (\hat{w} - \hat{r}) \lambda_{LS}$

⇒

(B25) $\lambda_{LS} \hat{S} + \lambda_{LC} \hat{C} = (\hat{w} - \hat{r})(\theta_{KC} \sigma_C \lambda_{LC} + \theta_{KS} \sigma_S \lambda_{LS}) + \hat{L}_0$

The corresponding equation for the capital market is:

(B26) $K_0 = \lambda_{KS} \hat{S} + \lambda_{KC} \hat{C} + \theta_{LS} \sigma_S (\hat{w} - \hat{r}) \lambda_{KS} + \theta_{LC} \sigma_C (\hat{w} - \hat{r}) \lambda_{KC}$

⇒

(B27) $\lambda_{KS} \hat{S} + \lambda_{KC} \hat{C} = -(\hat{w} - \hat{r})(\theta_{LS} \sigma_S \lambda_{KS} + \theta_{LC} \sigma_C \lambda_{KC}) + \hat{K}_0$

Subtracting (B27) from (B25) gives the final expression for the factor-market equilibrium:

(B28) $\lambda^*(\hat{S} - \hat{C}) = (\hat{w} - \hat{r})(\alpha_S \sigma_S + \alpha_C \sigma_C) + \hat{L}_0 - \hat{K}_0$

Where

(B29) $\lambda^* = \lambda_{LS} - \lambda_{KS} = \lambda_{KC} - \lambda_{LC}$

(B30) $\alpha_S = \theta_{KS} \lambda_{LS} + \theta_{LS} \lambda_{KS}$

(B31) $\alpha_C = \theta_{KC} \lambda_{LC} + \theta_{LC} \lambda_{KC}$

As long as $\lambda^* > 0$, i.e. the S sector is relatively labour intensive in production, there will be a positive relationship between $(S / K)$ and $(w / r)$. An increase in the labour force will give a positive shift in northeast direction, while increased capital will lead to a negative shift in south western direction.

To see the relationship between $\sigma_S$ and the more usual definition of the technical rate of substitution via the production function, the latter can be expressed by: $\sigma_S = -\frac{d \log(K / L)}{d \log(r / w)} = \frac{d \log K}{d \log(w / r)} - \frac{d \log L}{d \log(w / r)}$. 
**B2 Connection between factor prices and commodity prices**

With constant returns to scale and perfect competition, the commodity prices will be equal to the unit cost functions:

(B32) \( P_S = c_S(w, r) \)

(B33) \( P_C = c_C(w, r) \)

\[ \Rightarrow \]

(B34) \[ \hat{P}_S = \frac{w_{CS}}{c_S} \hat{w} + \frac{r_{KS}}{c_S} \hat{r} = \theta_{CS} \hat{w} + \theta_{KS} \hat{r} \]

(B35) \[ \hat{P}_C = \theta_{LC} \hat{w} + \theta_{KC} \hat{r} \]

\[ \Rightarrow \]

(B36) \[ \hat{P}_S - \hat{P}_C = \theta^* (\hat{w} - \hat{r}) \]

Where

(B37) \[ \theta^* = \theta_{LS} - \theta_{LC} = \theta_{KC} - \theta_{KS} \]

If \( \theta^* > 0 \) there will be a positive relationship between \( \frac{P_S}{P_C} \) and \( \frac{w}{r} \). This will be the case when S goods are relatively labour intensive in production, i.e. when \( \lambda^* > 0 \).

**B3 Demand side**

I will introduce a demand side with homothetic preferences:

(B38) \[ \hat{S} - \hat{C} = -\sigma_D (\hat{P}_S - \hat{P}_C) \]

\( \sigma_D \) is the elasticity of substitution on the demand side, \( \sigma_D > 0 \).

**B4 The relative price levels**

Combining all the relationships in the previous calculations gives the four-quadrant figure below, diagram B1. The first quadrant gives the demand relationship and the supply that will be derived from the factor-market equilibrium (4th quadrant) and the relationship between goods-prices and factor-prices (second quadrant). The third quadrant simply shows that the relative factor-prices in the second quadrant have to be equal to the relative prices in the fourth quadrant.
From the figure we see that a poor country with relatively less capital per labour unit, will have lower relative price level than the rich country in the autarky situation. In the integrated equilibrium, the two countries will have an equal price level, between the two autarky levels.

\[(B39) \quad \left( \frac{P_s}{P_c} \right)^D < \left( \frac{P_s}{P_c} \right)^{D\leftrightarrow O} < \left( \frac{P_s}{P_c} \right)^{O}\]

The integrated equilibrium price level \( \left( \frac{P_s}{P_c} \right)^{D\leftrightarrow O} \) can then be found either by using these equations and diagram B.1 or in a diagram for the two countries’ total production as done in diagram 7. With help of the indifference curves in diagram 7 it is also possible to find the consumption point.